

On the long-time asymptotics of quantum dynamical semigroups¹.

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Abstract:

We consider semigroups $\{\alpha_t : t \geq 0\}$ of normal, unital, positive maps α_t on a W^* -algebra \mathcal{M} . The (predual) semigroup $\nu_t(\rho) := \rho \circ \alpha_t$ on normal states ρ of \mathcal{M} leaves invariant the face $\mathcal{F}_p := \{\rho : \rho(p) = 1\}$ supported by the projection $p \in \mathcal{M}$, iff $\alpha_t(p) \geq p$ (i.e., p is subharmonic). We complete the arguments showing that the sub-harmonic projections form a complete lattice. We then consider r_o , the smallest projection which is larger than each support of a minimal invariant face; then r_o is subharmonic. In finite dimensional cases and when α_t is completely positive, $\sup \alpha_t(r_o) = \mathbf{1}$ and r_o is also the smallest projection p for which $\alpha_t(p) \rightarrow \mathbf{1}$. If $\{\nu_t : t \geq 0\}$ admits a faithful family of normal stationary states then $r_o = \mathbf{1}$ is useless; if not, it helps to reduce the problem of the asymptotic behaviour of the semigroup for large times.

Keywords: Quantum dynamical semigroups; sub-harmonic projections; long-time asymptotics.

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