

On the long-time asymptotics of quantum dynamical semigroups<sup>1</sup>.

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Abstract:

We consider semigroups  $\{\alpha_t : t \geq 0\}$  of normal, unital, positive maps  $\alpha_t$  on a  $W^*$ -algebra  $\mathcal{M}$ . The (predual) semigroup  $\nu_t(\rho) := \rho \circ \alpha_t$  on normal states  $\rho$  of  $\mathcal{M}$  leaves invariant the face  $\mathcal{F}_p := \{\rho : \rho(p) = 1\}$  supported by the projection  $p \in \mathcal{M}$ , iff  $\alpha_t(p) \geq p$  (i.e.,  $p$  is subharmonic). We complete the arguments showing that the sub-harmonic projections form a complete lattice. We then consider  $r_o$ , the smallest projection which is larger than each support of a minimal invariant face; then  $r_o$  is subharmonic. In finite dimensional cases and when  $\alpha_t$  is completely positive,  $\sup \alpha_t(r_o) = \mathbf{1}$  and  $r_o$  is also the smallest projection  $p$  for which  $\alpha_t(p) \rightarrow \mathbf{1}$ . If  $\{\nu_t : t \geq 0\}$  admits a faithful family of normal stationary states then  $r_o = \mathbf{1}$  is useless; if not, it helps to reduce the problem of the asymptotic behaviour of the semigroup for large times.

Keywords: Quantum dynamical semigroups; sub-harmonic projections; long-time asymptotics.

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